MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210)

BMS2024 – ADVANCED MANAGERIAL STATISTICS

(All sections / Groups)

11 FEBRUARY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **TEN (10)** printed pages inclusive of the cover page, formulae sheet and statistical tables.
2. Answer **ALL** four questions in answer booklet provided.
3. Students are allowed to use authorized calculators by lecturer only.
4. Marks are shown at the end of each question.

Question 1 [15 marks]

Numerous grocery stores have introduced self-checkout systems so that customers can scan and pay for their own purchases. The manager at Tesco would like to hear how customers feel about this service and how frequently they utilize it. Below is a six-day sample of the number of clients using the service at one of the Tesco branches.

120	118	91	104	108	97

Can we conclude that the mean number of customers using the self-checkout system is more than 100 per day? Given that population standard deviation is 25.

a) Determine the null and alternative hypothesis statement. [2 marks]
b) Compute the test statistics. [7 marks]
c) Compute the p-value of the test. [3 marks]
d) Make conclusion using the p-value in (c). [3 marks]

Question 2 [30 marks]

The administrator of a new program at College wants to estimate the GPA in a new program. He thought that that high school GPA (HSGPA), the reading score on the Scholastic Aptitude Test (READ) and math score on the SAT (MATH) would predict the GPA for the new program. The analysis based on nine students are given as follow:

ANOVA table:

	df	*SS*	*MS*
Regression		4.3595	
Residual/Error			0.1407
Total	8		

	Coefficient	*p-value*
Intercept	– 0.4111	0.622
HSGPA	1.2014	0.011
READ	0.0016	0.025
MATH	– 0.0019	0.461

a) Complete the ANOVA table. [5 marks]
b) Determine the coefficient of determination and interpret the value. [5 marks]
c) Determine the regression equation. [3 marks]
d) Test whether each of the factors has significant impact on GPA. [13 marks]
e) Interpret the impact of SAT reading score on GPA. [4 marks]

Continued...

Question 3 [30 marks]

The following data are the monthly tuition fees for 5-SPM subjects charged in three different states in Malaysia. The ANOVA table is presented as well.

Selangor	Malacca	Penang
200	180	250
220	190	225
250	200	185
255	205	200

ANOVA table:

	df	*SS*	*MS*
Among Groups			
Within Groups			
Total	11	7666.667	

Assuming the data to be Normally distributed, can we conclude that there is difference in the mean monthly tuition fees for various states at 5% significant level?

a) Determine the appropriate statistical technique for this study. State the reasons of your choice. [4 marks]
b) Compute $\overline{\overline{X}}$ and sample mean for each state. [8 marks]
c) Complete the ANOVA table. [9 marks]
d) Determine the null and alternative hypothesis statement. [2 marks]
e) Compute the test statistics. [2 marks]
f) Determine the critical value for the test. [2 marks]
g) Determine the conclusion for the test. [3 marks]

Question 4 [25 marks]

Each year the personnel department in a large corporation assesses the performance of all of its employees. Each employee is rated on the happiness working at the corporation on a 7-point scale where 1 = very unhappy and 7 = very happy. The president of the company believes that the female employees are less happy compared to the male employees. To examine the validity of this belief, she draws a random sample of six female employees' scores and another six male employees' scores last year.

Continued...

Employees Happiness Scores

Female	Male
2	5
2	5
4	4
4	3
3	4
4	3

a) What is the most appropriate statistical technique for this study? State the reasons of your choice. [4 marks]
b) Determine the null and alternative hypothesis statement. [2 marks]
c) Determine which one is the Group 1. [1 mark]
d) Compute the test statistics. [13 marks]
e) Determine the critical value for the test. [2 marks]
f) Determine the conclusion for the test. [3 marks]

End of Question...

STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Sample Mean = $\overline{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$

Sample Standard Deviation = $s = \sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{\left(\sum_{i=1}^{n} X_i\right)^2}{n(n-1)}}$

where n = *number of observations*
X_i = *the* i^{th} *observation of the data*

B. HYPOTHESIS TESTING

Types of Error

Type I Error = α = P(Rejecting H_0 | H_0 is true) where, Confidence Interval = 1 - α

Type II Error = β = P(Not Rejecting H_0 | H_0 is false)

One Sample Mean Test

σ Known	σ Unknown
$z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}}$	$t = \dfrac{\overline{x} - \mu}{s/\sqrt{n}}$

Two Sample Mean Test

Comparing Means for Two Independent Populations

[Standard Deviation (σ) Known]

$$z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

[Standard Deviation (σ) Not Known]

$$t = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$$

where $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$

Continued...

Two Sample Mean Test

Comparing Means for Two Paired Populations

$$t = \frac{\left(\overline{D} - \mu_D\right)}{S_D/\sqrt{n}} \quad \text{where } \overline{D} = \frac{\sum_{i=1}^{n} D_i}{n} \quad \text{and } S_D = \sqrt{\frac{\sum_{i=1}^{n} D_i^2}{n-1} - \frac{\left(\sum_{i=1}^{n} D_i\right)^2}{n(n-1)}}$$

C. ANALYSIS OF VARIANCE (ANOVA)

One-Way ANOVA

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F-statistic
Among Groups	*c - 1*	SSA	MSA = SSA/*(c-1)*	MSA/MSW
Within Groups	*n - c*	SSW	MSW = SSW/*(n-c)*	
Total	*n - 1*	SST		

$$SST = \sum_{j=1}^{c}\sum_{i=1}^{n_j}\left(X_{ij} - \overline{\overline{X}}\right)^2$$

or alternative formula:

$$SST = \left(\sum_{j=1}^{c}\sum_{i=1}^{n_i} X_{ij}^2\right) - \frac{\left(\sum_{j=1}^{c}\sum_{i=1}^{n_i} X_{ij}\right)^2}{n}$$

$$SSA = \sum_{j=1}^{c} n_j\left(\overline{X}_j - \overline{\overline{X}}\right)^2 \quad \text{and} \quad \text{SSW} = \text{SST} - \text{SSA}$$

where n = number of observations, c = number of groups and $\overline{\overline{X}}$ = overall mean

Tukey-Kramer Procedure

$$\text{Critical Range} = Q_U\sqrt{\frac{MSW}{2}\left[\frac{1}{n_i} + \frac{1}{n_j}\right]} \quad \text{where} \quad Q_u = Q_{\alpha\,;\,c,\,n-c}$$

Continued...

Two-Way ANOVA				
Source	**Degrees of Freedom**	**Sum of Squares**	**Mean Squares**	**F-statistic**
A	$r-1$	SSA	MSA = SSA/$(r-1)$	MSA / MSE
B	$c-1$	SSB	MSB = SSB/$(c-1)$	MSB / MSE
AB	$(r-1)(c-1)$	SSAB	MSAB = SSAB/$(r-1)(c-1)$	MSAB / MSE
Error	$rc\,(n-1)$	SSE	MSE = SSE/ $rc(n'-1)$	
Total	$n-1$	SST		

where,

Factor A levels are represented by the rows and Factor B levels are represented by the columns and
n = number of observations
c = number of columns
r = number of rows
n' = number of replicates

$$SST = \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n'}\left(X_{ijk} - \overline{\overline{X}}\right)^2 \qquad SSA = cn'\sum_{i=1}^{r}\left(\overline{X}_i - \overline{\overline{X}}\right)^2$$

$$SSB = rn'\sum_{j=1}^{c}\left(\overline{X}j - \overline{\overline{X}}\right)^2$$

where $\overline{\overline{X}}$ = *overall mean*

$$SSE = (n'-1)\left[S_1^2 + S_2^2 + \ldots\ldots + S_k^2\right]$$

where S_i^2 = variance of each block

D. NON-PARAMETRIC ANALYSIS

Non-Parametric Analysis	
Wilcoxon Rank Sum Test	**Wilcoxon Signed Rank Sum Test**
$Z = \dfrac{\left(T_1 - \mu_{T_1}\right)}{\sigma_{T_1}}$ where	$Z = \dfrac{\left(T_+ - \mu_{T_+}\right)}{\sigma_{T_+}}$ where
$\mu_{T1} = \dfrac{n_1(n+1)}{2}$ and	$\mu_{T+} = \dfrac{n(n+1)}{4}$ and
$\sigma_{T_1} = \sqrt{\dfrac{n_1 n_2 (n+1)}{12}}$ where $n = n_1 + n_2$	$\sigma_{T_+} = \sqrt{\dfrac{n(n+1)(2n+1)}{24}}$

Continued...

Kruskal-Wallis Rank Test

$$H = \left[\frac{12}{n(n+1)} \sum_{j=1}^{c} \frac{T_j^2}{n_j} \right] - 3(n+1)$$ where the critical value is χ^2 with $df = c - 1$

Check ranking sum: $\sum T_j = n(n+1)/2$

Chi-Square Test

$$\chi^2 = \sum^{n} \frac{(O-E)^2}{E}$$

where O = Frequency of Observed Values and E = Frequency of Expected Values

with df = c – 1 where c = number of categories
or
with df = (r – 1)(c – 1) where r = number of rows and c = number of columns

E. REGRESSION ANALYSIS

Multiple Linear Regression

Population Model: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots\ldots + \beta_k X_k + \varepsilon$

Sample Model: $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \ldots\ldots + b_k x_k + e$

Adjusted R-Square $= 1 - \left[\frac{(1-R^2)(n-1)}{(n-p-1)} \right]$

where p = number of independent/predictor variables

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	p	SSR	MSR = SSR/p
Error/Residual	$n-p-1$	SSE	MSE = SSE/$(n-p-1)$
Total	$n-1$	SST	

Test Statistic for Significance of the Overall Regression Model

F = MSR/MSE

Test Statistic for Significance of Each Predictor Variable

$t_i = \frac{b_i}{S_{b_i}}$ and the critical value $= \pm t_{\alpha/2,(n-p-1)}$

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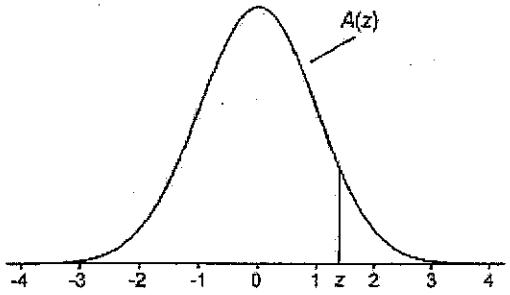
TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

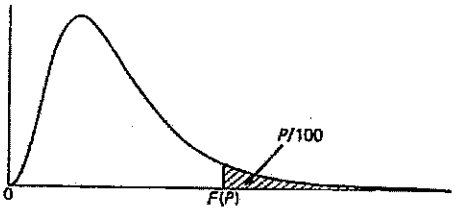
z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

Continued...

TABLE 12(*b*). 5 PER CENT POINTS OF THE *F*-DISTRIBUTION

If $F = \frac{X_1}{\nu_1} \Big/ \frac{X_2}{\nu_2}$, where X_1 and X_2 are independent random variables distributed as χ^2 with ν_1 and ν_2 degrees of freedom respectively, then the probabilities that $F \geqslant F(P)$ and that $F \leqslant F'(P)$ are both equal to $P/100$. Linear interpolation in ν_1 and ν_2 will generally be sufficiently accurate except when either $\nu_1 > 12$ or $\nu_2 > 40$, when harmonic interpolation should be used.



(This shape applies only when $\nu_1 \geqslant 3$. When $\nu_1 < 3$ the mode is at the origin.)

$\nu_1 =$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2 = 1$	161·4	199·5	215·7	224·6	230·2	234·0	236·8	238·9	241·9	243·9	249·1	254·3
2	18·51	19·00	19·16	19·25	19·30	19·33	19·35	19·37	19·40	19·41	19·45	19·50
3	10·13	9·552	9·277	9·117	9·013	8·941	8·887	8·845	8·786	8·745	8·639	8·526
4	7·709	6·944	6·591	6·388	6·256	6·163	6·094	6·041	5·964	5·912	5·774	5·628
5	6·608	5·786	5·409	5·192	5·050	4·950	4·876	4·818	4·735	4·678	4·527	4·365
6	5·987	5·143	4·757	4·534	4·387	4·284	4·207	4·147	4·060	4·000	3·841	3·669
7	5·591	4·737	4·347	4·120	3·972	3·866	3·787	3·726	3·637	3·575	3·410	3·230
8	5·318	4·459	4·066	3·838	3·687	3·581	3·500	3·438	3·347	3·284	3·115	2·928
9	5·117	4·256	3·863	3·633	3·482	3·374	3·293	3·230	3·137	3·073	2·900	2·707
10	4·965	4·103	3·708	3·478	3·326	3·217	3·135	3·072	2·978	2·913	2·737	2·538
11	4·844	3·982	3·587	3·357	3·204	3·095	3·012	2·948	2·854	2·788	2·609	2·404
12	4·747	3·885	3·490	3·259	3·106	2·996	2·913	2·849	2·753	2·687	2·505	2·296
13	4·667	3·806	3·411	3·179	3·025	2·915	2·832	2·767	2·671	2·604	2·420	2·206
14	4·600	3·739	3·344	3·112	2·958	2·848	2·764	2·699	2·602	2·534	2·349	2·131
15	4·543	3·682	3·287	3·056	2·901	2·790	2·707	2·641	2·544	2·475	2·288	2·066
16	4·494	3·634	3·239	3·007	2·852	2·741	2·657	2·591	2·494	2·425	2·235	2·010
17	4·451	3·592	3·197	2·965	2·810	2·699	2·614	2·548	2·450	2·381	2·190	1·960
18	4·414	3·555	3·160	2·928	2·773	2·661	2·577	2·510	2·412	2·342	2·150	1·917
19	4·381	3·522	3·127	2·895	2·740	2·628	2·544	2·477	2·378	2·308	2·114	1·878
20	4·351	3·493	3·098	2·866	2·711	2·599	2·514	2·447	2·348	2·278	2·082	1·843
21	4·325	3·467	3·072	2·840	2·685	2·573	2·488	2·420	2·321	2·250	2·054	1·812
22	4·301	3·443	3·049	2·817	2·661	2·549	2·464	2·397	2·297	2·226	2·028	1·783
23	4·279	3·422	3·028	2·796	2·640	2·528	2·442	2·375	2·275	2·204	2·005	1·757
24	4·260	3·403	3·009	2·776	2·621	2·508	2·423	2·355	2·255	2·183	1·984	1·733
25	4·242	3·385	2·991	2·759	2·603	2·490	2·405	2·337	2·236	2·165	1·964	1·711
26	4·225	3·369	2·975	2·743	2·587	2·474	2·388	2·321	2·220	2·148	1·946	1·691
27	4·210	3·354	2·960	2·728	2·572	2·459	2·373	2·305	2·204	2·132	1·930	1·672
28	4·196	3·340	2·947	2·714	2·558	2·445	2·359	2·291	2·190	2·118	1·915	1·654
29	4·183	3·328	2·934	2·701	2·545	2·432	2·346	2·278	2·177	2·104	1·901	1·638
30	4·171	3·316	2·922	2·690	2·534	2·421	2·334	2·266	2·165	2·092	1·887	1·622
32	4·149	3·295	2·901	2·668	2·512	2·399	2·313	2·244	2·142	2·070	1·864	1·594
34	4·130	3·276	2·883	2·650	2·494	2·380	2·294	2·225	2·123	2·050	1·843	1·569
36	4·113	3·259	2·866	2·634	2·477	2·364	2·277	2·209	2·106	2·033	1·824	1·547
38	4·098	3·245	2·852	2·619	2·463	2·349	2·262	2·194	2·091	2·017	1·808	1·527
40	4·085	3·232	2·839	2·606	2·449	2·336	2·249	2·180	2·077	2·003	1·793	1·509
60	4·001	3·150	2·758	2·525	2·368	2·254	2·167	2·097	1·993	1·917	1·700	1·389
120	3·920	3·072	2·680	2·447	2·290	2·175	2·087	2·016	1·910	1·834	1·608	1·254
∞	3·841	2·996	2·605	2·372	2·214	2·099	2·010	1·938	1·831	1·752	1·517	1·000

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